

Main Ideas

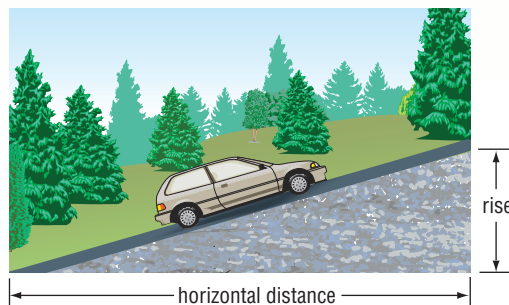
- Find and use the slope of a line.
- Graph parallel and perpendicular lines.

New Vocabulary

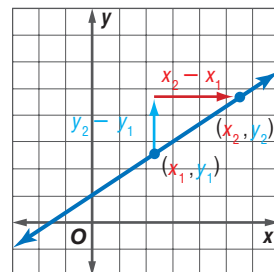
rate of change
slope
family of graphs
parent graph
oblique

GET READY for the Lesson

The grade of a road is a percent that measures the steepness of the road. It is found by dividing the amount the road rises by the corresponding horizontal distance.



Slope A **rate of change** measures how much a quantity changes, on average, relative to the change in another quantity, often time. The idea of rate of change can be applied to points in the coordinate plane to determine the steepness of the line between the points. The **slope** of a line is the ratio of the change in y -coordinates to the corresponding change in x -coordinates. Suppose a line passes through points at (x_1, y_1) and (x_2, y_2) .



$$\begin{aligned} \text{slope} &= \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}} \\ &= \frac{y_2 - y_1}{x_2 - x_1} \end{aligned}$$

KEY CONCEPT**Slope of a Line**

Words The slope of a line is the ratio of the change in y -coordinates to the change in x -coordinates.

Symbols The slope m of the line passing through (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$.

Study Tip**Slope**

The formula for slope is often remembered as *rise over run*, where the rise is the difference in y -coordinates and the run is the difference in x -coordinates.

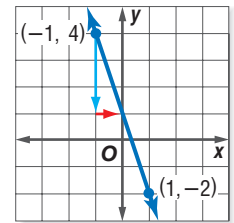
EXAMPLE Find Slope and Use Slope to Graph

- 1** Find the slope of the line that passes through $(-1, 4)$ and $(1, -2)$. Then graph the line.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Slope formula} \\ &= \frac{-2 - 4}{1 - (-1)} && (x_1, y_1) = (-1, 4), (x_2, y_2) = (1, -2) \\ &= \frac{-6}{2} \text{ or } -3 && \text{The slope is } -3. \end{aligned}$$

(continued on the next page)

Graph the two ordered pairs and draw the line. Use the slope to check your graph by selecting any point on the line. Then go down 3 units and right 1 unit or go up 3 units and left 1 unit. This point should also be on the line.



CHECK Your Progress

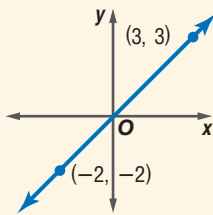
- Find the slope of the line that passes through $(1, -3)$ and $(3, 5)$. Then graph the line.

The slope of a line tells the direction in which it rises or falls.

CONCEPT SUMMARY

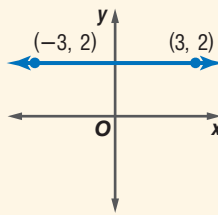
Slope

If the line rises to the right, then the slope is *positive*.



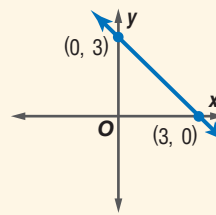
$$m = \frac{3 - (-2)}{3 - (-2)} = 1$$

If the line is horizontal, then the slope is *zero*.



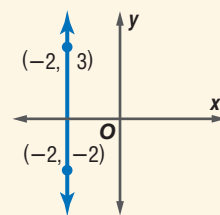
$$m = \frac{2 - 2}{3 - (-3)} = 0$$

If the line falls to the right, then the slope is *negative*.



$$m = \frac{0 - 3}{3 - 0} = -1$$

If the line is vertical, then the slope is *undefined*.



$x_1 = x_2$, so m is undefined.

Study Tip

Slope is Constant

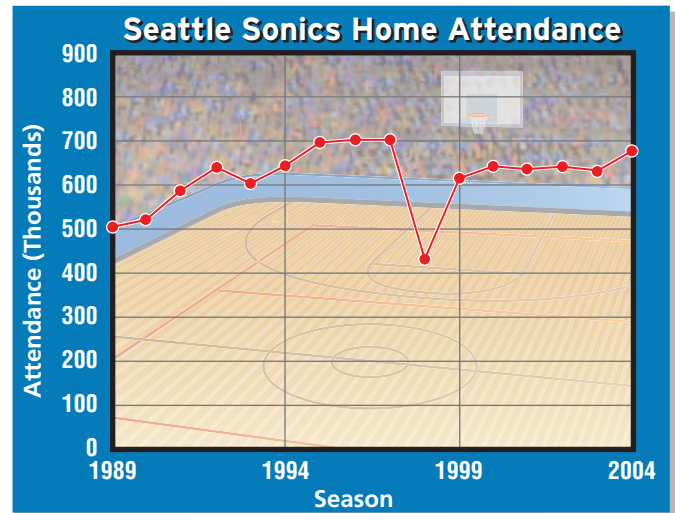
The slope of a line is the same, no matter what two points on the line are used.

Real-World EXAMPLE

- BASKETBALL** Refer to the graph at the right. Find the rate of change of the number of people attending Seattle Sonics home games from 1993 to 1996.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Slope formula} \\ &= \frac{700 - 601}{1996 - 1993} && \text{Substitute.} \\ &\approx 33 && \text{Simplify.} \end{aligned}$$

Between 1993 and 1996, the number of people attending Seattle Sonics home games increased at an average rate of about 33(1000) or 33,000 people per year.



Source: Kenn.com

CHECK Your Progress

- In 1999, 45,616 students applied for admission to UCLA. In 2004, 56,878 students applied. Find the rate of change in the number of students applying for admission from 1999 to 2004.

Parallel and Perpendicular Lines A family of graphs is a group of graphs that displays one or more similar characteristics. The parent graph is the simplest of the graphs in a family.

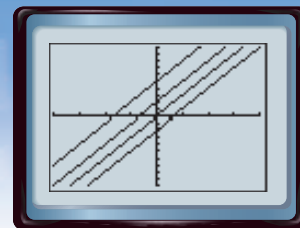
GRAPHING CALCULATOR LAB

Lines with the Same Slope

The calculator screen shows the graphs of $y = 3x$, $y = 3x + 2$, $y = 3x - 2$, and $y = 3x + 5$.

THINK AND DISCUSS

1. What is similar about the graphs? What is different about the graphs?
2. Write another function that has the same characteristics as these graphs. Check by graphing.



$[-4, 4]$ scl: 1 by $[-10, 10]$ scl: 1

In the Lab, you saw that lines that have the same slope are parallel.

Study Tip

Horizontal Lines

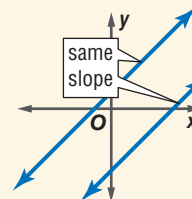
All horizontal lines are parallel because they all have a slope of 0.

KEY CONCEPT

Parallel Lines

Words In a plane, nonvertical lines with the same slope are parallel. All vertical lines are parallel.

Model



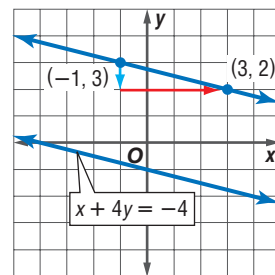
EXAMPLE Parallel Lines

- 3** Graph the line through $(-1, 3)$ that is parallel to the line with equation $x + 4y = -4$.

The x -intercept is -4 , and the y -intercept is -1 . Use the intercepts to graph $x + 4y = -4$.

The line falls 1 unit for every 4 units it moves to the right, so the slope is $-\frac{1}{4}$.

Now use the slope and the point at $(-1, 3)$ to graph the line parallel to the graph of $x + 4y = -4$.



CHECK Your Progress

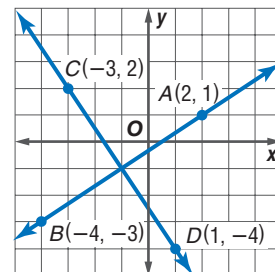
- 3.** Graph the line through $(-2, 4)$ that is parallel to the line with equation $x - 3y = 3$.

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The graphs of \overleftrightarrow{AB} and \overleftrightarrow{CD} are perpendicular.

$$\begin{array}{l} \text{slope of line } AB \\ \frac{-3 - 1}{-4 - 2} = \frac{-4}{-6} \text{ or } \frac{2}{3} \end{array} \qquad \begin{array}{l} \text{slope of line } CD \\ \frac{-4 - 2}{1 - (-3)} = \frac{-6}{4} \text{ or } -\frac{3}{2} \end{array}$$

The slopes are opposite reciprocals of each other. The product of the slopes of two perpendicular lines is always -1 .



Reading Math

Oblique

An oblique line is a line that is neither horizontal nor vertical.

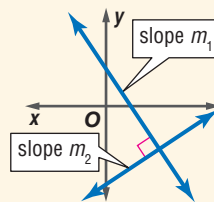
KEY CONCEPT

Perpendicular Lines

Words In a plane, two oblique lines are perpendicular if and only if the product of their slopes is -1 .

Symbols Suppose m_1 and m_2 are the slopes of two oblique lines. Then the lines are perpendicular if and only if $m_1 m_2 = -1$, or $m_1 = -\frac{1}{m_2}$.

Model



Any vertical line is perpendicular to any horizontal line.

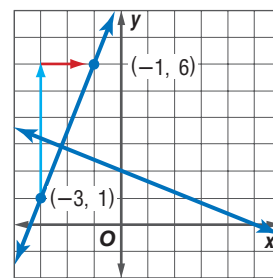
EXAMPLE Perpendicular Lines

- 1 Graph the line through $(-3, 1)$ that is perpendicular to the line with equation $2x + 5y = 10$.

The x -intercept is 5, and the y -intercept is 2. Use the intercepts to graph $2x + 5y = 10$.

The line falls 2 units for every 5 units it moves to the right, so the slope is $-\frac{2}{5}$. The slope of the perpendicular line is the opposite reciprocal of $-\frac{2}{5}$, or $\frac{5}{2}$.

Start at $(-3, 1)$ and go up 5 units and right 2 units. Use this point and $(-3, 1)$ to graph the line.



CHECK Your Progress

4. Graph the line through $(-6, 2)$ that is perpendicular to the line with equation $3x - 2y = 6$.

CHECK Your Understanding

Example 1 (pp. 71–72)

Find the slope of the line that passes through each pair of points.

1. $(-2, -1), (2, -3)$ 2. $(2, 2), (4, 2)$ 3. $(4, 5), (-1, 0)$

Graph the line passing through the given point with the given slope.

4. $(2, -1), -3$ 5. $(-3, -4), \frac{3}{2}$

Example 2 (p. 72)

WEATHER For Exercises 6–8, use the table that shows the temperatures at different times on the same day.

Time	8:00 A.M.	10:00 A.M.	12:00 P.M.	2:00 P.M.	4:00 P.M.
Temp (°F)	36	47	55	58	60

6. What was the average rate of change of the temperature from 8:00 A.M. to 10:00 A.M.?
7. What was the average rate of change of the temperature from 12:00 P.M. to 4:00 P.M.?
8. During what 2-hour period was the average rate of change of the temperature the least?

Example 3
(p. 73)

Example 4
(p. 74)

Graph the line that satisfies each set of conditions.

9. passes through $(0, 3)$, parallel to graph of $6y - 10x = 30$
10. passes through $(1, 1)$ parallel to graph of $x + y = 5$
11. passes through $(4, -2)$, perpendicular to graph of $3x - 2y = 6$
12. passes through $(-1, 5)$, perpendicular to graph of $5x - 3y - 3 = 0$

Exercises

HOMEWORK HELP	
For Exercises	See Examples
13–24	1
25–29	2
30–37	3, 4

Find the slope of the line that passes through each pair of points.

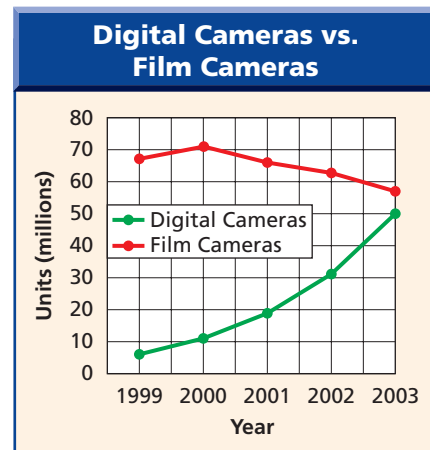
13. $(4, -1), (6, -6)$
14. $(-8, -3), (2, 3)$
15. $(8, 7), (7, -6)$
16. $(-2, -3), (0, -5)$
17. $(4, 9), (11, 9)$
18. $(4, -1.5), (4, 4.5)$

Graph the line passing through the given point with the given slope.

19. $(-1, 4), m = \frac{2}{3}$
20. $(-3, -1), m = -\frac{1}{5}$
21. $(3, -4), m = 2$
22. $(1, 2), m = -3$
23. $(6, 2), m = 0$
24. $(-2, -3)$, undefined

CAMERAS For Exercises 25 and 26, refer to the graph that shows the number of digital still cameras and film cameras sold in recent years.

25. Find the average rate of change of the number of digital cameras sold from 1999 to 2003.
26. Find the average rate of change of the number of film cameras sold from 1999 to 2003. What does the sign of the rate mean?



Source: Digital Photography Review

TRAVEL For Exercises 27–29, use the following information.

Mr. and Mrs. Wellman are taking their daughter to college. The table shows their distance from home after various amounts of time.

Time (h)	Distance (mi)
0	0
1	55
2	110
3	165
4	165

27. Find the average rate of change of their distance from home between 1 and 3 hours after leaving home.
28. Find the average rate of change of their distance from home between 0 and 5 hours after leaving home.
29. What is another word for *rate of change* in this situation?

Graph the line that satisfies each set of conditions.

30. passes through $(-2, 2)$, parallel to a line whose slope is -1
31. passes through $(2, -5)$, parallel to graph of $x = 4$
32. passes through origin, parallel to graph of $x + y = 10$
33. passes through $(2, -1)$, parallel to graph of $2x + 3y = 6$
34. passes through $(2, -1)$, perpendicular to graph of $2x + 3y = 6$
35. passes through $(-4, 1)$, perpendicular to a line whose slope is $-\frac{3}{2}$
36. passes through $(3, 3)$, perpendicular to graph of $y = 3$
37. passes through $(0, 0)$, perpendicular to graph of $y = -x$

Find the slope of the line that passes through each pair of points.

38. $(\frac{1}{2}, -\frac{1}{3}), (\frac{1}{4}, \frac{2}{3})$

39. $(\frac{1}{2}, \frac{2}{3}), (\frac{5}{6}, \frac{1}{4})$

40. $(c, 5), (c, -2)$

41. $(3, d), (-5, d)$

42. **WASHINGTON MONUMENT** The Washington Monument, in Washington, D.C., is 555 feet $5\frac{1}{8}$ inches tall and weighs 90,854 tons. The monument is topped by a square aluminum pyramid. The sides of the pyramid's base measure 5.6 inches, and the pyramid is 8.9 inches tall. Estimate the slope that a face of the pyramid makes with its base.



43. Determine the value of r so that the line through $(5, r)$ and $(2, 3)$ has slope 2.

44. Determine the value of r so that the line through $(6, r)$ and $(9, 2)$ has slope $\frac{1}{3}$.

Graph the line that satisfies each set of conditions.

45. perpendicular to graph of $3x - 2y = 24$, intersects that graph at its x -intercept

46. perpendicular to graph of $2x + 5y = 10$, intersects that graph at its y -intercept

47. **GEOMETRY** Determine whether quadrilateral $ABCD$ with vertices $A(-2, -1)$, $B(1, 1)$, $C(3, -2)$, and $D(0, -4)$ is a rectangle. Explain.

For Exercises 48 and 49, use a graphing calculator to investigate the graphs of each set of equations. Explain how changing the slope affects the graph of the line.

48. $y = 2x + 3, y = 4x + 3, y = 8x + 3, y = x + 3$

49. $y = -3x + 1, y = -x + 1, y = -5x + 1, y = -7x + 1$



Graphing Calculator

H.O.T. Problems

50. **OPEN ENDED** Write an equation of a line with slope 0. Describe the graph of the equation.

51. **CHALLENGE** If the graph of the equation $ax + 3y = 9$ is perpendicular to the graph of the equation $3x + y = -4$, find the value of a .

52. **FIND THE ERROR** Gabriel and Luisa are finding the slope of the line through $(2, 4)$ and $(-1, 5)$. Who is correct? Explain your reasoning.

$$m = \frac{\text{Gabriel}}{5-4} \text{ or } \frac{1}{2-(-1)}$$

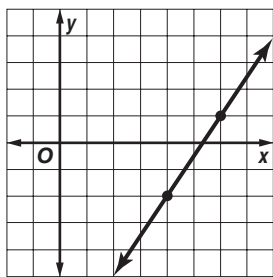
$$m = \frac{\text{Luisa}}{4-5} \text{ or } -\frac{1}{2-(-1)}$$

53. **REASONING** Determine whether the statement *A line has a slope that is a real number is sometimes, always, or never true.* Explain your reasoning.

54. **Writing in Math** Use the information about the grade of a road on page 71 to explain how slope applies to the steepness of roads. Include a graph of $y = 0.08x$, which corresponds to a grade of 8%.

EXTRA PRACTICE
See pages 893, 927.
Math online
Self-Check Quiz at algebra2.com

55. **ACT/SAT** What is the slope of the line shown in the graph?



- A $-\frac{3}{2}$
 B $-\frac{2}{3}$
 C $\frac{2}{3}$
 D $\frac{3}{2}$

56. **REVIEW** The table below shows the cost of bananas depending on the amount purchased. Which conclusion can be made based on information in the table?

Cost of Bananas	
Number of Pounds	Cost (\$)
5	1.45
20	4.60
50	10.50
100	19.00

- F The cost of 10 pounds of bananas would be more than \$4.00.
 G The cost of 200 pounds of bananas would be at most \$38.00.
 H The cost of bananas is always more than \$0.20 per pound.
 J The cost of bananas is always less than \$0.28 per pound.

Spiral Review

Find the x -intercept and the y -intercept of the graph of each equation. Then graph the equation. (Lesson 2-2)

57. $-2x + 5y = 20$

58. $4x - 3y + 8 = 0$

59. $y = 7x$

Find each value if $f(x) = 3x - 4$. (Lesson 2-1)

60. $f(-1)$

61. $f(3)$

62. $f\left(\frac{1}{2}\right)$

63. $f(a)$

Solve each inequality. (Lessons 1-5 and 1-6)

64. $5 < 2x + 7 < 13$

65. $2z + 5 \geq 1475$

66. **SCHOOL** A test has multiple-choice questions worth 4 points each and true-false questions worth 3 points each. Marco answers 14 multiple-choice questions correctly. How many true-false questions must he answer correctly to get at least 80 points total? (Lesson 1-5)

Simplify. (Lessons 1-1 and 1-2)

67. $\frac{1}{3}(15a + 9b) - \frac{1}{7}(28b - 84a)$

68. $3 + (21 \div 7) \times 8 \div 4$

GET READY for the Next Lesson

PREREQUISITE SKILL Solve each equation for y . (Lesson 1-3)

69. $x + y = 9$

70. $4x + y = 2$

71. $-3x - y + 7 = 0$

72. $5x - 2y - 1 = 0$

73. $3x - 5y + 4 = 0$

74. $2x + 3y - 11 = 0$